STATE-SPACE MODELING AND CONTROL OF A LEVITATING TRAIN

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Introduction

Trains are one of the primary means of land transport in the world, as they are energy efficient and reach high speeds compared to their counterparts. However, Conventional trains waste considerable energy due to the friction between the wheels and the track. To reduce this energy loss, Magnetic Levitation Trains were invented, where compartments are elevated above the tracks with no physical contact using magnetism principles.

Due to the nature of this system and its dynamics, it is not fully stable on its own. Therefore, to successfully implement this levitation idea on the trains, numerous control techniques are used. There are three main dynamics in this system which can be decoupled for simplicity. First is levitation, where an electromagnet is used either to repel or attract the train body to levitate it above tracks. Second is propulsion, where linear induction motors are used to propel the train. Third is guidance, where opposite pole magnets on the sides keep the train centered in the lateral direction. Figure 1 below shows all three dynamics with respect to the train compartments:

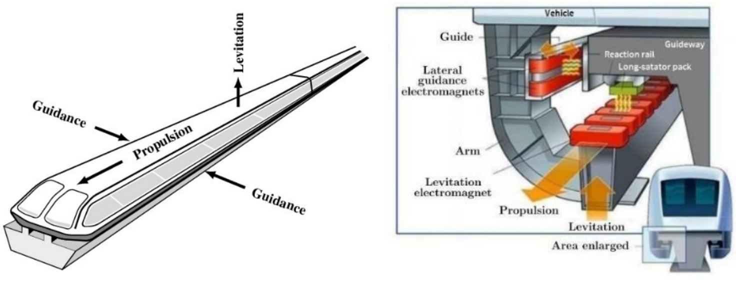

Figure 1. Dynamics of a Maglev Train [1][2]

In this project, researchers focused on the levitation dynamics (vertical axis). In this axis, an electromagnet is attached to the undercarriage of the train and a permanent magnet to the track;

an upward magnetic force is generated to levitate the compartment; this force is dependent on the input current and the distance between the magnets. Using a specific current input, the compartment can be levitated precisely above a certain height. Subsequent sections discuss this single-input single-output (SISO) system's modeling, analysis of properties, controller design and simulation, and conclusions and discussions.

Modeling

The first step in modeling this system is representing it as a free-body diagram (FBD). Figure 2 shows the FBD of the system; there are two forces in the vertical direction: the force of gravity (F_g=mg) and the opposing magnetic force (F_m). F_m is given by this formula:

$$F_m = \frac{N^2 I^2 \mu_\circ A}{4Z^2} = \frac{KI^2}{Z^2}, \quad \text{where K} = \frac{N^2 \mu_\circ A}{4} \qquad [3]$$

N → No of turns in electromagnet coil

$\mu_\circ$ → Permeability of free space

I → Current flowing through electromagnet coil

A → Effective Area of the magnet pole face

Z → Air gap distance between train and guideway

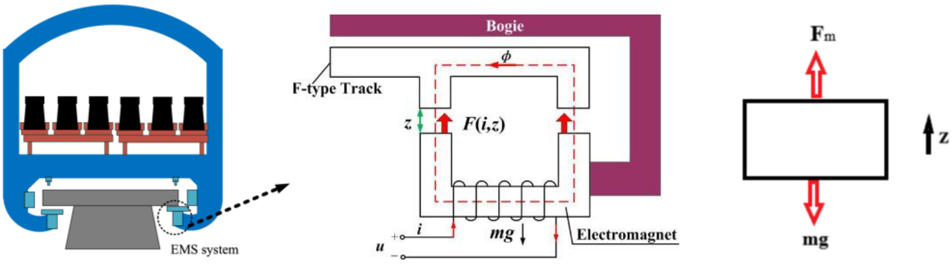

Figure 2. Levitation Axis FBD [3]

Equations of motion were derived from the FBD using Newton's second law:

$$\sum F = \text{ma}; \qquad \text{m}\ddot{z} = \frac{KI^2}{Z} - \text{mg}$$

Subsequently, the state space model was constructed by taking: X_1 = Z, $X_2 = \dot{Z}$, & input u = I

$$\dot{X} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} X_2 \\ \frac{Ku^2}{mX_1^2} - g \end{bmatrix} = \begin{matrix} f_1(X_1, X_2, u) \\ f_1(X_1, X_2, u) \end{matrix}$$

Since the resulting state-space representation is highly non-linear, the system was linearized about an equilibrium levitation point using Taylor series expansion:

$$\dot{\tilde{X}} = f(\tilde{X}, \tilde{u}) = 0, \qquad \text{Therefore, } \tilde{X}_2 = 0$$

$$\frac{K\tilde{u}^2}{m\tilde{X}_1^2} - \text{g} = 0 \qquad \rightarrow \qquad \tilde{u}^2 = \frac{m\tilde{X}_1^2}{K}$$

$$\text{Let } \tilde{u} = I_\circ, \qquad \tilde{X} = \begin{bmatrix} \tilde{X}_1 \\ 0 \end{bmatrix}, \qquad \text{Let } \tilde{X}_1 = Z_\circ$$

$I_\circ^2 = \frac{mgZ_\circ^2}{K}$ (Relation between equilibrium current $I_\circ$ and air gap $Z_\circ$)

$$\text{A} = \begin{bmatrix} \frac{\partial f_1}{\partial X_1} & \frac{\partial f_1}{\partial X_2} \\ \frac{\partial f_2}{\partial X_1} & \frac{\partial f_2}{\partial X_2} \end{bmatrix}_{(\tilde{u}, \tilde{X})} \qquad \text{B} = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \end{bmatrix}_{(\tilde{u}, \tilde{X})}$$

$$\dot{X}_\delta = \underbrace{\begin{bmatrix} 0 & 1 \\ \frac{-2KI_\circ^2}{mZ_\circ^3} & 0 \end{bmatrix}}_{A} X_\delta + \underbrace{\begin{bmatrix} 0 \\ \frac{2KI_\circ}{mZ_\circ^2} \end{bmatrix}}_{B} \text{u}, \quad y_\delta = \underbrace{[1 \quad 0]}_{C} X_\delta + \underbrace{[0]}_{D} \text{u}$$

Analysis of properties

After the sate-space modeling, important properties of the system, including controllability, observation, internal stability, and BIBO stability, were determined as follows:

Controllability

$$\text{P} = [B \quad AB]$$

$$\text{P} = [B \quad AB] = \begin{bmatrix} 0 & \frac{2KI_\circ}{mZ_\circ^2} \\ \frac{2KI_\circ}{mZ_\circ^2} & 0 \end{bmatrix}$$

$$\det(\text{P}) = -\frac{4K^2I_\circ^2}{m^2Z_\circ^4} \neq 0$$

Observability

$$\text{Q} = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$\text{Q} = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(\text{Q}) = 1 \neq 0$$

Since the determinant of both P and Q matrices is not equal to zero, the system is both controllable and observable. The internal stability of the system was determined using Lyapunov's indirect method, and the BIBO stability was determined using the roots of the transfer function as follows:

Lyapunov's Indirect Method → check the real part of the eigenvalues of system matrix A

$|\lambda - \text{I A}| = 0 \quad \rightarrow \quad \begin{vmatrix} \lambda & -1 \\ \frac{2KI_\circ^2}{mZ_\circ^3} & \lambda \end{vmatrix} = 0 \quad \rightarrow \quad \lambda = \pm j\sqrt{\frac{2KI_\circ^2}{mZ_\circ^3}}$, (A.M. = G.M. = 1) for each λ

Real part of eigenvalue is zero → The linearized model is marginally stable → Also, the stability of the actual non-linear system is unknown by this method.

BIBO Stability: The open Loop transfer function G(s) for the state space system is given by:

$$G(s) = C\,(sI - A)^{-1}B + D$$

$$G(s) = C\,(sI - A)^{-1}B = [1 \quad 0] \cdot \frac{1}{s^2 + \frac{2KI_\circ^2}{mz_\circ^3}} \begin{bmatrix} s & 1 \\ -\frac{2KI_\circ^2}{mz_\circ^3} & s \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \frac{2KI_\circ}{mz_\circ^2} \end{bmatrix} = \frac{\frac{2KI_\circ}{mz_\circ^2}}{s^2 + \frac{2KI_\circ^2}{mz_\circ^3}}$$

$$s = \pm j\sqrt{\frac{2KI_\circ^2}{mz_\circ^3}}$$

Since the transfer function's poles have no real parts, the system is not BIBO stable.

Controller Design and Simulation Results

Since the linearized system is marginally stable, there needs to be a state feedback control that actively controls the system and stabilizes it, given certain design specifications. As learned in the coursework, a PD controller was chosen. The first step in designing a PD controller is determining the open loop dynamic response of the system, which is as follows:

$$\text{Open loop transfer function } G(s) = \frac{\frac{2KI_\circ}{mz_\circ^2}}{s^2 + \frac{2KI_\circ^2}{mz_\circ^3}},\ y_{final} = G(0) = \frac{Z_\circ}{I_\circ}$$

Standard form, G(s) = $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$, ζ = 0, $\omega_n = \sqrt{\frac{2KI_\circ^2}{mz_\circ^3}}$

<u>Percent Overshoot:</u> %OS = $e^{\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)} \times$ 100% = $e^0 \times$ 100% = 100%

<u>Settling Time:</u> $T_s = \frac{4}{\zeta\omega_n}$ = ∞ (system never settles)

<u>For closed loop system</u>: a PD controller was designed: u = - KX + Gr , where K = $[K_p \quad K_d]$

$$\dot{X} = (A - BK)\,X + BGr \quad \rightarrow \quad \dot{X} = \begin{bmatrix} 0 & 1 \\ -\left(\frac{2KI_\circ^2}{mZ_\circ^3}\right) - \left(\frac{2KI_\circ}{mZ_\circ^2}\right)K_p & -\left(\frac{2KI_\circ}{mZ_\circ^2}\right)K_d \end{bmatrix} X + \begin{bmatrix} 0 \\ \left(\frac{2KI_\circ}{mZ_\circ^2}\right) G \end{bmatrix} r$$

$$y = (C - DK)\,X + DGr \quad \rightarrow \quad y = [1 \quad 0]X + [0]r$$

Closed Loop characteristic polynomial:

$$|sI - (A - BK)| = \begin{bmatrix} s & -1 \\ \left(\frac{2KI_\circ^2}{mZ_\circ^3}\right) + \left(\frac{2KI_\circ}{mZ_\circ^2}\right)K_p & s + \left(\frac{2KI_\circ}{mZ_\circ^2}\right)K_d \end{bmatrix} = s^2 + \left(\left(\frac{2KI_\circ}{mZ_\circ^2}\right)K_d\right)s + \left(\frac{2KI_\circ^2}{mZ_\circ^3}\right) + \left(\frac{2KI_\circ}{mZ_\circ^2}\right)K_p$$

on comparing with the desired characteristic polynomial

$$2\zeta_{des}\omega_{n_{des}} = \left(\frac{2KI_\circ}{mZ_\circ^2}\right)K_d \rightarrow K_d = \frac{\zeta_{des}\omega_{n_{des}}}{\frac{KI_\circ}{mZ_\circ^2}}; \qquad \omega_n^2 = \left(\frac{2KI_\circ^2}{mZ_\circ^3}\right) + \left(\frac{2KI_\circ}{mZ_\circ^2}\right)K_p \rightarrow K_p = \frac{\omega_{n_{des}}^2 - \frac{2KI_\circ^2}{mZ_\circ^3}}{\frac{2KI_\circ}{mZ_\circ^2}}$$

Using real life values: m = 2250 Kg, K = 0.01, $Z_\circ$ = 0.04 m, & formula $\left(I_\circ^2 = \frac{mZ_\circ^2}{K}\right) \rightarrow I_\circ$ = 60 A

<u>Design Specifications for controller:</u> settling time t_s = 0.24 sec, percent overshoot = 4 %

$$\zeta = \sqrt{\frac{\left(ln\frac{\%\,O.S.}{100}\right)^2}{\pi^2 + \left(ln\frac{\%O.S.}{100}\right)^2}} = 0.716 \qquad \omega_n = \frac{4}{0.24 \times 0.716} = 23.277$$

$$K_d = \frac{\zeta\omega_n}{\frac{KI_\circ}{mZ_\circ^2}} = 99.99 \qquad K_p = \frac{\omega_n^2 - \frac{2KI_\circ^2}{mZ_\circ^3}}{\frac{2KI_\circ}{mZ_\circ^2}} = 126.789$$

$$y_{final}^{OL} = \frac{Z_\circ}{I_\circ} = \frac{0.04}{60} = 6.67 \times 10^{-4}$$

$$y_{final}^{CL} = y_{final}^{OL} = H_{CL}(0) = \text{-}C(A - BK)^{-1}BG \quad \rightarrow \quad G = \frac{y_{final}^{OL}}{-C(A-BK)^{-1}B} = 1.095$$

Using the values obtained above in a MATLAB code (added in the appendix), figure 3 shows the dynamic response of the open-loop vs closed-loop system, given a unit step input:

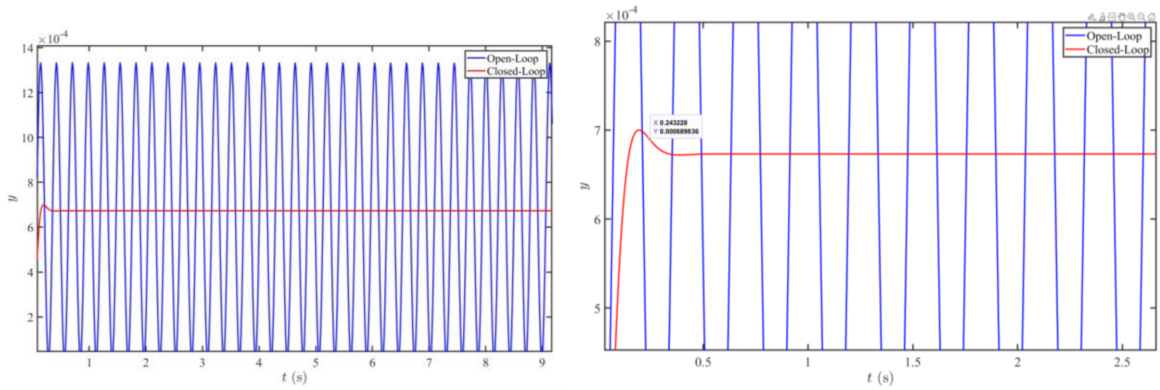

Figure 3. Dynamic response of the open-loop vs closed-loop system

Conclusions and discussions

A MagLev train's vertical dynamics result in a non-linear system. Upon linearizing, the linearized system is controllable, observable, and marginally stable. The open-loop system was marginally stable (oscillating about the steady state value). A PD Controller was used to stabilize the linearized system asymptotically, and realistic design parameters from relevant studies were used. MATLAB Simulation does verify that the system was stabilized according to the parameters (0.24 second settling time and 4% overshoot). A major challenge in executing this project was determining the stability of the original non-linear system, as methods discussed in the class could not conclusively determine if the original non-linear system was marginally stable or unstable. Major learnings include modeling a real-world dynamic system to state space form, examining its properties, and stabilizing it using a controller. Next Steps would be exploring PID controllers that account for steady-state error caused by changes in overall mass of the train. Also, exploring and controlling the other two dynamics of the MagLev train.

Contributions of team members

Vishavjit Singh Khinda (50%): Individually Modeled the system and derived state-space equations, co-created the paper and PPT.

Sukhpreet Singh Nolastname (50%): Individually Modeled the system and derived state-space equations, co-created the paper and PPT.

References

[1] T. D. F. Cabral and F. R. Chavarette, “DYNAMICS AND CONTROL DESIGN VIA LQR AND SDRE METHODS FOR a MAGLEV SYSTEM,” *International Journal of Pure and Apllied Mathematics*, vol. 101, no. 2, pp. 289–300, Jan. 2015, doi: 10.12732/ijpam.v101i2.13.

[2] “Genetic algorithm tuned super twisting sliding mode controller for suspension of Maglev train with flexible track,” *IEEE Journals & Magazine | IEEE Xplore*, 2023. https://ieeexplore.ieee.org/document/10082920

[3] M. Zhai, Z. Long, and X. Li, “Calculation and evaluation of load performance of magnetic levitation system in medium-low speed maglev train,” *International Journal of Applied Electromagnetics and Mechanics*, vol. 61, no. 4, pp. 519–536, Jun. 2019, doi: 10.3233/jae-190031.

Appendix: MATLAB Code

```
close all; clc;
% System parameters
m = 2250;
k = 0.01;
Z_not = 0.04;
I_not = 60;

% Controller gains
G = 1.095;
k_d = 99.99;
k_p = 126.789;

% Plot of Open loop vs closed loop System
s = tf('s');
H_OL = ((2*k*I_not)/(m*Z_not^2))/(s^2 + (2*k*I_not^2)/(m*Z_not^3));
[y_OL,t1]=step(H_OL,10);
H_CL = ((G*2*k*I_not)/(m*Z_not^2))/(s^2 + (2*k*I_not*k_d*s)/(m*Z_not^2) +
(2*k*I_not^2)/(m*Z_not^3) + (2*k*I_not*k_p)/(m*Z_not^2));
[y_CL,t2]=step(H_CL,10);

figure
plot(t1,y_OL,'b','LineWidth',2)
hold
plot(t2,y_CL,'r','LineWidth',2)
xlabel('$t$ (s)', 'Interpreter','latex')
ylabel('$y$', 'Interpreter','latex')
legend('Open-Loop','Closed-Loop')
set(gca,'linewidth',2,'fontsize',20,'fontname','Times');
set(gcf,'color','white')
```